

Higgs Chaotic Inflation in Standard Model and NMSSM

Kazunori Nakayama^(a) and Fuminobu Takahashi^(b)^(a) *Theory Center, KEK, 1-1 Oho, Tsukuba, Ibaraki 305-0801, Japan*^(b) *Institute for the Physics and Mathematics of the Universe, University of Tokyo, Chiba 277-8583, Japan*

(Dated: September 28, 2010)

We construct a chaotic inflation model in which the Higgs fields play the role of the inflaton in the standard model as well as in the singlet extension of the supersymmetric standard model. The key idea is to allow a non-canonical kinetic term for the Higgs field. The model is a realization of the recently proposed running kinetic inflation, in which the coefficient of the kinetic term grows as the inflaton field. The inflaton potential depends on the structure of the Higgs kinetic term. In the simplest cases, the inflaton potential is proportional to ϕ^2 and $\phi^{2/3}$ in the standard model and NMSSM, respectively. It is also possible to have a flatter inflaton potential.

PACS numbers: 98.80.Cq

The inflation is strongly motivated by the recent WMAP results [1]. It is a non-trivial task to construct a successful inflation model, partly because the properties of the inflaton are poorly known. The inflaton may be only weakly coupled to the standard model (SM) sector. In this case, since the number of cosmological observables are limited, it might be difficult to pin down the inflation model even with the Planck data [2]. Alternatively, the inflaton may be a part of the SM or its extensions [3–5], in which case we may be able to study the properties of the inflaton at collider experiments such as the LHC. The latter idea has recently attracted much attention since the proposal of the SM Higgs inflation [6]. In the model of Ref. [6], the flat potential is achieved by introducing a non-minimal coupling to the gravity [7] (see also Refs. [8–13] for the inflation with non-minimal coupling to gravity in supergravity). In this letter we pursue another approach to the inflation in the SM and its extensions: we construct a Higgs chaotic inflation model by allowing a non-canonical kinetic term. As we shall see below, the model is a realization of the running kinetic inflation [14, 15].

Recently, a new class of inflation models was proposed by one of the authors (FT) [14], in which the kinetic term grows as the inflaton field, making the effective potential flat [16, 17]. This model naturally fits with a high-scale inflation model such as chaotic inflation [18], in which the inflaton moves over a Planck scale or even larger within the last 50 or 60 e-foldings [19]. This is because the precise form of the kinetic term may well change after the inflaton travels such a long distance. In some cases, the change could be so rapid, that it significantly affects the inflaton dynamics. We named such model as running kinetic inflation. Interestingly, the power of the inflaton potential generically changes in this class of inflation models. The phenomenological aspects of the running kinetic inflation was studied in detail in Ref. [15].

First let us consider the Higgs inflation in the SM. In order to identify the Higgs with the inflaton, there are two issues. First, if the potential were valid up to large field values, the chaotic inflation with a quartic potential would occur. However, the quartic chaotic inflation is

strongly disfavored by observation [1]. Secondly, in order to satisfy the WMAP normalization, a quartic coupling must be as small as $O(10^{-13})$ which would result in an unacceptably light Higgs mass. These issues can be avoided if the potential becomes flatter at large field values. There are two ways. One is to introduce a non-minimal coupling to gravity [7] and the other is to make use of the running kinetic term [14]. We will focus on the latter possibility in this letter.

The key idea is to add the following interaction,

$$\Delta\mathcal{L} = \xi|H|^2|D_\mu H|^2, \quad (1)$$

where H is the Higgs doublet, ξ is a numerical coefficient, and D_μ denotes a gauge covariant derivative¹. Here and in what follows we adopt the Planck unit, $M_P = 1$. In the unitary gauge, we can write down the Lagrangian for the Higgs h :

$$\mathcal{L} = \frac{1}{2} \left(1 + \xi \frac{h^2}{2} \right) (\partial h)^2 - \frac{\lambda_h}{4} (h^2 - v^2)^2. \quad (2)$$

For small h , the effect of non-canonical kinetic term is irrelevant, while, for large $h \gtrsim 1/\sqrt{\xi}$, the kinetic term grows, that is why the name “running kinetic inflation.” The canonically normalized field in this regime is given by

$$\hat{h} \approx \frac{\sqrt{\xi} h^2}{2\sqrt{2}}, \quad (3)$$

and the effective potential becomes

$$V(\hat{h}) \simeq \frac{1}{2} \left(\frac{4\lambda_h}{\xi} \right) \hat{h}^2. \quad (4)$$

Thus, the quadratic chaotic inflation occurs. We emphasize here that the potential changes from h^4 to \hat{h}^2 because

¹ In Ref. [20] a different kind of non-canonical kinetic term was considered.

of the running kinetic term. A large kinetic term makes the effective potential flatter, and it is straightforward to obtain a flatter potential by increasing the power of h in the coefficient of the kinetic term. The WMAP normalization gives $\lambda_h \simeq 10^{-11}\xi$, and so, if ξ is sufficiently large, $\xi \sim O(10^{10})$, the quartic coupling λ_h can be of $O(0.1)$. Such a large coupling is analogous to the non-minimal coupling to gravity in Ref. [6], and it may be obtained by tuning or some UV dynamics [16, 17]. Note that the inflation takes place for sub-Planckian values of h , while the value of \hat{h} exceeds the Planck scale.

Next we apply the same idea to the Higgs inflation in supergravity. As is well known, it is difficult to implement the chaotic inflation in supergravity because of the exponential pre-factor e^K in the scalar potential, where K is the Kähler potential. In order to construct a chaotic inflation model in supergravity, there must be flat directions in the field space along which the Kähler potential does not grow. The flat direction can be realized by either symmetry or tuning. In the latter case we can assume that a certain interaction in the Kähler potential is enhanced, which results in an approximate flat direction [17, 21]. Instead, we here adopt the symmetry to ensure the flatness, following the construction in Refs. [14, 15]. In both cases, the inflation dynamics is essentially the same. In the pioneering paper by Kawasaki, Yamaguchi and Yanagida [22], a shift symmetry on the inflaton, $\phi \rightarrow \phi + \alpha$ (α is a real transformation parameter), was introduced so that the Kähler potential depends only on $(\phi - \phi^\dagger)$, not on $(\phi + \phi^\dagger)$. In Ref. [14], the shift symmetry is generalized to $\phi^n \rightarrow \phi^n + \alpha$, based on the idea that the form of the kinetic term may change after the inflaton traverses more than the Planck scale.

Let us now construct a Higgs inflation model in the singlet extension of MSSM. We introduce a chiral superfield, ϕ , to represent the gauge invariant $H_u H_d$:

$$\phi^2 \equiv H_u H_d, \quad (5)$$

where H_u and H_d are the up- and down-type Higgs superfields. In the scalar components, we can express

$$H_u = \begin{pmatrix} 0 \\ \phi \end{pmatrix}, \quad H_d = \begin{pmatrix} \phi \\ 0 \end{pmatrix}. \quad (6)$$

We require that the Kähler potential for the Higgs fields is invariant under the following transformation;

$$\phi^2 \rightarrow \phi^2 + \alpha \quad (7)$$

where α is a real transformation parameter. This corresponds to the above-mentioned shift symmetry with $n = 2$. We will discuss the case of another value of n later. The symmetry (7) means that the composite field $\hat{\phi} \sim \phi^2$ transforms under a Nambu-Goldstone like shift symmetry.

The Kähler potential satisfying the shift symmetry (7) must be a function of $(\phi^2 - \phi^{\dagger 2})$:

$$K = \sum_{\ell=1} \frac{c_\ell}{\ell} (\phi^2 - \phi^{\dagger 2})^\ell \quad (8)$$

where c_ℓ is a numerical coefficient of $O(1)$ and we normalize $c_2 \equiv -1$; c_ℓ is real (imaginary) for even(odd) ℓ . Note that the $|H_u|^2$ and $|H_d|^2$ terms are absent. Instead, the kinetic term for ϕ arises from the terms of $\ell \geq 2$, whose contribution is proportional to $(\phi^2 - \phi^{\dagger 2})^{\ell-2} |\phi|^2$. One can show that $(\phi^2 - \phi^{\dagger 2})$ remains constant along the inflationary trajectory by noting that $(\phi^2 - \phi^{\dagger 2})$ appears explicitly in the Kähler potential and therefore acquires a large mass during inflation [14]. So, we drop terms with $\ell \geq 3$ because it does not change the form of the kinetic term.

We can impose a discrete Z_k symmetry that is consistent with the shift symmetry (7). Requiring $(\phi^2 - \phi^{\dagger 2})$, an invariant under the shift symmetry, be also invariant under the discrete symmetry up to a phase factor, we find that k must be either 2 or 4. If $k = 4$, the $(\phi^2 - \phi^{\dagger 2})$ would flip its sign, and so, c_ℓ with any odd ℓ should vanish. If $k = 2$, there is no such constraint, since ϕ^2 itself is invariant under Z_2 .

In order to have a successful inflation, we introduce explicit symmetry breaking terms in both the Kähler and super-potentials:

$$K = \kappa |\phi|^2 - \frac{1}{2} (\phi^2 - \phi^{\dagger 2})^2 + |X|^2, \quad (9)$$

$$W = \lambda X \phi^2, \quad (10)$$

where the κ - and λ -terms are the symmetry breaking terms, and we assume $\kappa, \lambda \ll 1$. There could be other symmetry breaking terms, but we assume that they are soft in a sense that the shift symmetry remains a good symmetry at least up to the inflaton field value of $O(10)$. Here X is a singlet superfield. X can be stabilized at the origin during and after inflation if we add $-a|X|^4$ in the Kähler potential with $a = O(1)$. The presence of X not only simplifies the inflaton potential, but also helps to avoid a situation that the inflaton potential becomes negative due to $-3|W|^2$ in the scalar potential [22]. Here and in what follows we impose Z_4 symmetry under which X and ϕ^2 flip the sign, in order to suppress dangerous couplings such as $\int d^2\theta X$. The charge assignment of X and ϕ are shown in Table I.

It may be instructive to write down explicitly the Kähler and super-potentials in terms of H_u and H_d :

$$K = \kappa_u |H_u|^2 + \kappa_d |H_d|^2 - \frac{1}{2} (H_u H_d - (H_u H_d)^\dagger)^2 + |X|^2, \quad (11)$$

$$W = \lambda X H_u H_d, \quad (12)$$

with $\kappa_u + \kappa_d = \kappa$. We note that the form of the superpotential (12) is equivalent to the part of the interactions in NMSSM.

The scalar potential in supergravity is given by

$$V = e^K \left(D_i W K^{i\bar{j}} (D_{\bar{j}} W)^* - 3|W|^2 \right). \quad (13)$$

Since we have imposed the Z_4 symmetry, $\phi^2 - \phi^{\dagger 2} \approx$

	H_u	H_d	ϕ^2	X
$U(1)_R$	0	0	0	2
Z_4	1	1	2	2

TABLE I: The charge assignment of ϕ and X in the ϕ^2 chaotic inflation.

0 along the inflationary trajectory. The relevant Lagrangian for the inflation is then given by

$$\mathcal{L} = (\kappa + 2^2|\phi|^2) \partial^\mu \phi^\dagger \partial_\mu \phi - V(\phi), \quad (14)$$

$$V(\phi) \approx e^{\kappa|\phi|^2} \lambda^2 |\phi|^4. \quad (15)$$

Since we explicitly break the shift symmetry (7) by the κ term, there appears a non-vanishing exponential prefactor. However, for $|\phi| < 1/\sqrt{\kappa}$, the exponential prefactor is close to unity, and therefore can be dropped. Note that the inflaton does slow-roll even if the exponential pre-factor gives a main contribution to the tilt of the potential, as long as κ is much smaller than unity. Except for the exponential factor, one can see that ξ and λ_h in Eq. (2) are related to κ and λ as $\xi = 4/\kappa^2$ and $\lambda_h = \lambda^2/\kappa^2$.

For $1 < |\phi| \ll \kappa^{-1/2}$, the Lagrangian can be approximated by

$$\mathcal{L} \approx 2^2|\phi|^2 \partial^\mu \phi^\dagger \partial_\mu \phi - \lambda^2 |\phi|^4, \quad (16)$$

$$= \partial^\mu \hat{\phi}^\dagger \partial_\mu \hat{\phi} - \lambda^2 |\hat{\phi}|^2, \quad (17)$$

where we have defined $\hat{\phi} \equiv \phi^2$. The inflationary trajectory is given by $\phi^2 = \hat{\phi}^{\dagger 2}$, and so, the imaginary component of $\hat{\phi}$ vanishes. Let us rewrite the inflaton as

$$\hat{\phi} = \frac{\varphi}{\sqrt{2}}, \quad (18)$$

where φ is a real scalar. The Lagrangian for the canonically normalized inflaton is therefore given by

$$\mathcal{L} \approx \frac{1}{2} \partial^\mu \varphi \partial_\mu \varphi - \frac{\lambda^2}{2} \varphi^2, \quad (19)$$

for $1 < \varphi \ll \kappa^{-1}$. Thus, thanks to the shift symmetry, the inflaton φ can take a value greater than the Planck scale, and the chaotic inflation takes place.

The inflaton field during inflation is related to the e-folding number N as

$$\varphi_N \simeq \sqrt{4N}, \quad (20)$$

and the inflation ends at $\varphi \approx 1$. The power spectrum of the density perturbation is given by

$$\Delta_{\mathcal{R}}^2 \simeq \frac{V^3}{12\pi^2 V'^2} = (2.43 \pm 0.11) \times 10^{-9}, \quad (21)$$

where we have used in the second equality the WMAP result [1]. The coupling λ is therefore determined as

$$\lambda \simeq 8 \times 10^{-6} \left(\frac{N}{50}\right)^{-\frac{1}{2}} \simeq 2 \times 10^{13} \text{ GeV} \left(\frac{N}{50}\right)^{-\frac{1}{2}}. \quad (22)$$

In order for the inflation driven by (19) to last for N e-foldings, the following inequality must be met;

$$\varphi_N \lesssim \kappa^{-1} \iff \kappa \lesssim 0.07 \left(\frac{N}{50}\right)^{\frac{1}{2}}. \quad (23)$$

The spectral index n_s and the tensor-to-scalar ratio r are respectively given by

$$n_s = 1 - \frac{2}{N}, \quad (24)$$

$$r = \frac{8}{N}. \quad (25)$$

For $N = 50 \sim 60$, they vary as $n_s = 0.96 \sim 0.967$ and $r = 0.13 \sim 0.16$.

The inflation ends when the slow-roll condition is violated at $\varphi \sim 1$, and the inflaton starts to oscillate about the origin. The dynamics of the inflaton is then described by a complex scalar field $\hat{\phi}$ rather than the real scalar φ . As the amplitude of the inflaton decreases, the κ term becomes more important. For $|\phi| < (\kappa/4)^{1/2}$, the Lagrangian becomes

$$\mathcal{L} \approx \partial^\mu \tilde{\phi}^\dagger \partial_\mu \tilde{\phi} - \frac{\lambda^2}{\kappa^2} |\tilde{\phi}|^4, \quad (26)$$

where we have defined a canonically normalized field at low scales, $\tilde{\phi} \equiv \sqrt{\kappa} \phi$. Note that the power of the scalar potential changes from 2 to 4 after inflation. When the amplitude becomes of the order of the weak scale, the description by the D-flat direction $H_u H_d$ is no longer valid, and we should consider the dynamics of H_u and H_d separately as usual. In the end, they should develop vacuum expectation values (VEVs), leading to the electroweak phase transition.

In order to have a successful electroweak phase transition, the μ -term with a right magnitude must be generated. We may add small explicit breaking of the discrete symmetry to produce a tadpole of X , which makes X to develop a VEV, generating the μ -term. Alternatively, we may identify the X field as the singlet field in the NMSSM, in which the superpotential takes the following form,

$$W = \lambda X H_u H_d + y \frac{X^3}{3}, \quad (27)$$

where y is a coupling constant. Note that the presence of X^3 in the superpotential does not destabilize the inflation dynamics. In order to have a chaotic inflation in NMSSM, we need to consider a different shift symmetry. Instead of the Z_4 symmetry, let us assign Z_3 symmetry on X and the Higgs field.² See Table II. The simplest

² If the Z_3 is exact, domain walls will be produced. To avoid the domain-wall problem we need to introduce a small Z_3 breaking.

shift symmetry consistent with the Z_3 symmetry is given by³

$$\phi^6 \rightarrow \phi^6 + \alpha. \quad (28)$$

Along the same line, we can realize a chaotic inflation with the Higgs fields $H_u H_d$ as the inflaton. The Kähler potential is given by

$$K = c_1 (\phi^6 - \phi^{\dagger 6}) - \frac{1}{2} (\phi^6 - \phi^{\dagger 6})^2 + \dots, \quad (29)$$

where c_1 is in general non-zero. The potential is given by

$$V(\varphi) \approx \lambda^2 \left(\frac{\varphi}{\sqrt{2}} \right)^{\frac{2}{3}}, \quad (30)$$

where $\varphi = \sqrt{2}(\phi^6 - c_1/2)$ is the canonically normalized field. The spectral index and the tensor-to-scalar ratio, respectively, are $n_s = 0.973 \sim 0.978$ and $r = 0.044 \sim 0.053$ for $N = 50 \sim 60$. The WMAP normalization gives $\lambda \simeq 2 \times 10^{-5}$.

Note that, while $\lambda \sim 10^{-5}$ is determined by the WMAP normalization, the low-energy effective coupling between the singlet X and the Higgs is given by

$$\hat{\lambda} = \frac{\lambda}{\sqrt{\kappa_u \kappa_d}}. \quad (31)$$

So, if $\lambda \sim \kappa_u \sim \kappa_d$, the effective coupling can be $O(0.1)$. Similarly, the SM Yukawa interactions break the shift symmetry:

$$W_{MSSM} = y_u Q \bar{u} H_u + y_d Q \bar{d} H_d + y_e L \bar{e} H_d, \quad (32)$$

where $|y_{u,d,e}| \ll 1$ are the Yukawa couplings, and we suppressed the generations. The physical Yukawa couplings at the low energy are similarly scaled as

$$y_u^{(phys)} = \frac{y_u}{\sqrt{\kappa_u}}, \quad y_d^{(phys)} = \frac{y_d}{\sqrt{\kappa_d}}, \quad y_e^{(phys)} = \frac{y_e}{\sqrt{\kappa_d}}, \quad (33)$$

Therefore the top Yukawa coupling can be close to 1, if $y_u \sim \sqrt{\kappa_u} = O(10^{-3})$. The coefficients of the breaking terms are suppressed by a factor of $O(10^{-3})$ wherever either H_u or H_d appears: $y_u^{(top)} \sim 10^{-3}$, and $\lambda \sim \kappa_u \sim 10^{-6} - 10^{-5}$. This structure might be related with the UV theory behind the shift symmetry.

We emphasize here that the presence of X is essential for constructing a chaotic inflation model in supergravity. It is stabilized at the origin and its dynamics is not relevant for the inflation, and so, X may be considered as a spectator field. Interestingly, however, in the Higgs

Z_3	H_u	H_d	ϕ^2	X
	1	1	2	1

TABLE II: The charge assignment of ϕ and X in the $\phi^{2/3}$ chaotic inflation.

chaotic inflation model, the same X plays an important role in low-energy phenomenology. For instance, in the NMSSM, the fermionic superpartner of X can be dark matter.

So far we have considered the possibility that the Higgs fields play the role of the inflation. It is straightforward to apply the above idea to the other flat directions in MSSM. In this case we need to adopt a flat direction which is lifted by the superpotential of the form $W \sim X \phi^m$ [23]. (For instance the LLe direction could be lifted by $W = H_u L L e$, and X is identified with H_u). In particular, if the flat direction has a non-zero baryon/lepton number, the baryon/lepton numbers would be explicitly violated by the interactions in the Kähler potential, and so, the baryogenesis a la Affleck-Dine [24, 25] is possible [14, 15]. There would be no baryonic isocurvature perturbation [26] because the degree of the freedom orthogonal to the inflaton is heavy during inflation.

Let us briefly mention the reheating in the Higgs inflation model. The SM particles are naturally created by the inflaton decay in the Higgs inflation, but the process could be complicated by the non-perturbative decay. In the SM Higgs inflation and the NMSSM Higgs inflation with the Z_4 symmetry, the inflaton passes near the origin after inflation, and so, the preheating is likely to occur [27, 28]. On the other hand, in the last example, the inflaton acquires a non-zero angular momentum due to the non-zero c_1 . Then the preheating may not be efficient. If the non-perturbative decay is efficient in the former case, the resultant reheating temperature would be very high, and too many gravitinos may be produced from thermal scattering, while the non-thermal gravitino production [29, 30] is generically suppressed in the Higgs inflation.

If the Higgs chaotic inflation is realized in nature, we will be able to study the properties of the inflaton, namely the Higgs fields, at the collider experiments as well as the CMB observation.

Acknowledgments

This work was supported by the Grant-in-Aid for Scientific Research on Innovative Areas (No. 21111006) [KN and FT] and Scientific Research (A) (No. 22244030) [FT], and JSPS Grant-in-Aid for Young Scientists (B) (No. 21740160) [FT]. This work was supported by World Premier International Center Initiative (WPI Program), MEXT, Japan.

³ We can also consider a shift symmetry $\phi^{6\ell} \rightarrow \phi^{6\ell} + \alpha$ with $\ell = 2, 3, \dots$. The potential would be proportional to $\varphi^{2/3\ell}$ where

$$\varphi \sim \phi^{6\ell}.$$

-
- [1] E. Komatsu *et al.*, [arXiv:1001.4538 [astro-ph.CO]].
 - [2] [Planck Collaboration], arXiv:astro-ph/0604069.
 - [3] H. Murayama, H. Suzuki, T. Yanagida *et al.*, Phys. Rev. Lett. **70**, 1912-1915 (1993).
 - [4] S. Kasuya, T. Moroi, F. Takahashi, Phys. Lett. **B593**, 33-41 (2004). [hep-ph/0312094].
 - [5] R. Allahverdi, K. Enqvist, J. Garcia-Bellido *et al.*, Phys. Rev. Lett. **97**, 191304 (2006). [hep-ph/0605035].
 - [6] F. L. Bezrukov, M. Shaposhnikov, Phys. Lett. **B659**, 703-706 (2008). [arXiv:0710.3755 [hep-th]].
 - [7] D. S. Salopek, J. R. Bond, J. M. Bardeen, Phys. Rev. **D40**, 1753 (1989); T. Futamase, K. -i. Maeda, Phys. Rev. **D39**, 399-404 (1989); B. L. Spokoiny, Phys. Lett. **B147**, 39-43 (1984); R. Fakir, W. G. Unruh, Phys. Rev. **D41**, 1783-1791 (1990); E. Komatsu, T. Futamase, Phys. Rev. **D58**, 023004 (1998). [astro-ph/9711340].
 - [8] M. B. Einhorn and D. R. T. Jones, JHEP **1003**, 026 (2010) [arXiv:0912.2718 [hep-ph]].
 - [9] H. M. Lee, JCAP **1008**, 003 (2010). [arXiv:1005.2735 [hep-ph]].
 - [10] S. Ferrara, R. Kallosh, A. Linde *et al.*, Phys. Rev. **D82**, 045003 (2010). [arXiv:1004.0712 [hep-th]].
 - [11] S. Ferrara, R. Kallosh, A. Linde *et al.*, [arXiv:1008.2942 [hep-th]].
 - [12] R. Kallosh, A. Linde, [arXiv:1008.3375 [hep-th]].
 - [13] I. Ben-Dayan and M. B. Einhorn, arXiv:1009.2276 [hep-ph].
 - [14] F. Takahashi, Phys. Lett. B **693**, 140 (2010) [arXiv:1006.2801 [hep-ph]].
 - [15] K. Nakayama and F. Takahashi, arXiv:1008.2956 [hep-ph].
 - [16] S. Dimopoulos, S. D. Thomas, Phys. Lett. **B573**, 13-19 (2003). [hep-th/0307004].
 - [17] K. -I. Izawa, Y. Shinbara, [arXiv:0710.1141 [hep-ph]].
 - [18] A. D. Linde, Phys. Lett. B **129**, 177 (1983).
 - [19] D. H. Lyth, Phys. Rev. Lett. **78**, 1861 (1997).
 - [20] C. Germani and A. Kehagias, Phys. Rev. Lett. **105**, 011302 (2010) [arXiv:1003.2635 [hep-ph]].
 - [21] K. Nakayama and F. Takahashi, in preparation.
 - [22] M. Kawasaki, M. Yamaguchi, T. Yanagida, Phys. Rev. Lett. **85**, 3572-3575 (2000).
 - [23] T. Gherghetta, C. F. Kolda, S. P. Martin, Nucl. Phys. **B468**, 37-58 (1996). [hep-ph/9510370].
 - [24] I. Affleck and M. Dine, Nucl. Phys. B **249**, 361 (1985).
 - [25] M. Dine, L. Randall, S. D. Thomas, Nucl. Phys. **B458**, 291-326 (1996). [hep-ph/9507453].
 - [26] S. Kasuya, M. Kawasaki and F. Takahashi, JCAP **0810**, 017 (2008) [arXiv:0805.4245 [hep-ph]].
 - [27] F. Bezrukov, D. Gorbunov and M. Shaposhnikov, JCAP **0906**, 029 (2009) [arXiv:0812.3622 [hep-ph]].
 - [28] J. Garcia-Bellido, D. G. Figueroa and J. Rubio, Phys. Rev. D **79**, 063531 (2009) [arXiv:0812.4624 [hep-ph]].
 - [29] M. Kawasaki, F. Takahashi, T. T. Yanagida, Phys. Lett. **B638**, 8 (2006); Phys. Rev. D **74**, 043519 (2006).
 - [30] M. Endo, F. Takahashi, T. T. Yanagida, Phys. Lett. **B658**, 236 (2008); Phys. Rev. **D76**, 083509 (2007).